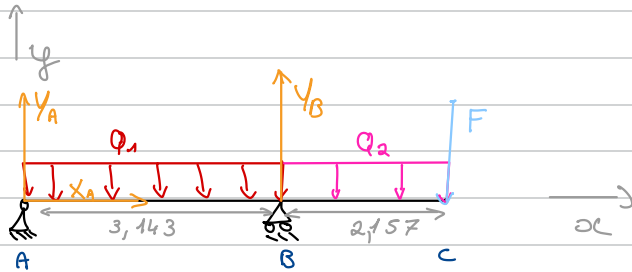


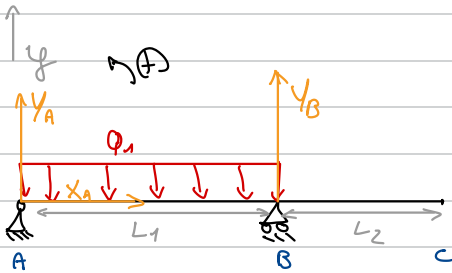
SAE MS1: groupe n° 8

Schéma mécanique:



Cas n° 1:

Schéma mécanique:



$$\begin{aligned} q_1 &= 20 \text{ kN/m} \\ L_1 &= 3,143 \text{ m} \\ L_2 &= 2,157 \end{aligned}$$

Degré d'hyperstatéité $\Rightarrow m = (2 + 1) - 3 = 0 \Rightarrow$ système isostatique

On applique le PFS sur la poutre AC au point A.

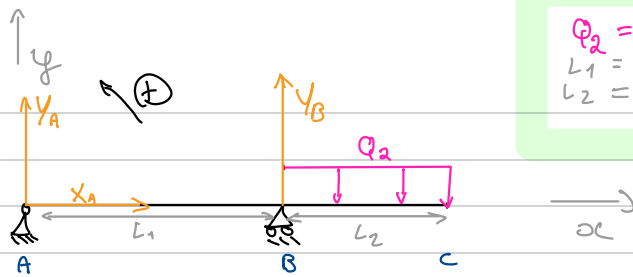
$$\begin{cases} X_A = 0 \\ Y_A + Y_B - q_1 \times L_1 = 0 \\ Y_B \times L_1 - q_1 \times L_1 \times \frac{L_1}{2} = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} X_A = 0 \\ Y_A = q_1 \times L_1 - Y_B \\ Y_B \times L_1 = q_1 \times \frac{L_1^2}{2} \end{cases} \Leftrightarrow \begin{cases} X_A = 0 \\ Y_A = q_1 \times \frac{L_1}{2} \\ Y_B = q_1 \times \frac{L_1}{2} \end{cases}$$

AN: \Leftrightarrow

$$\begin{cases} X_A = 0 \text{ N} \\ Y_A = 31,43 \text{ kN} \\ Y_B = 31,43 \text{ kN} \end{cases}$$

Cas n° 2:



$$q_2 = 16 \text{ kN/m}$$

$$L_1 = 3,143 \text{ m}$$

$$L_2 = 2,157 \text{ m}$$

Degré d'hyperstaticeité : $m = (2+1) - 3 = 0$

On applique le PFS sur la poutre AC au point A.

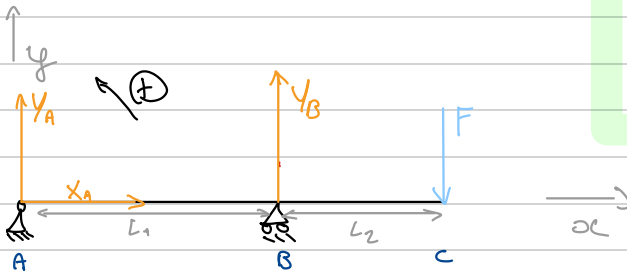
$$\begin{cases} X_A = 0 \\ Y_A + Y_B - q_2 \times L_2 = 0 \\ Y_B \times L_1 - q_2 \times L_2 \times \left(L_1 + \frac{L_2}{2} \right) = 0 \end{cases} \Leftrightarrow \begin{cases} X_A = 0 \\ Y_A = q_2 \times L_2 - Y_B \\ Y_B \times L_1 = q_2 \times L_2 \left(\frac{2L_1 + L_2}{2} \right) \end{cases}$$

$$\Leftrightarrow \begin{cases} X_A = 0 \\ Y_A = q_2 L_2 - q_2 \frac{L_2}{L_1} \left(\frac{2L_1 + L_2}{2} \right) = q_2 \frac{L_2^2}{2L_1} \\ Y_B = q_2 \times \frac{L_2}{L_1} \times \left(\frac{2L_1 + L_2}{2} \right) = \frac{2L_1 L_2 q_2 + L_2^2 q_2}{2L_1} \end{cases}$$

AN

$$\Leftrightarrow \begin{cases} X_A = 0 \text{ N} \\ Y_A = 11,84 \text{ kN} \\ Y_B = 46,35 \text{ kN} \end{cases}$$

Cas n° 3:



$$F = 8 \text{ kN}$$

$$L_1 = 3,143 \text{ m}$$

$$L_2 = 2,157 \text{ m}$$

Degré d'hyperstaticeité : $m = (2+1) - 3 = 0$

On applique le PFS sur la poutre AC au point A.

$$\begin{cases} X_A = 0 \\ Y_A + Y_B - F = 0 \\ Y_B \times L_1 - F \times (L_1 + L_2) = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} X_A = 0 \\ Y_A = F - Y_B \\ Y_B = F \times \left(\frac{L_1 + L_2}{L_1} \right) \end{cases}$$

$$\Leftrightarrow \begin{cases} X_A = 0 \\ Y_A = F \left(1 - \left(\frac{L_1 + L_2}{L_1} \right) \right) = -F \frac{L_2}{L_1} \\ Y_B = \frac{F \times (L_1 + L_2)}{L_1} \end{cases}$$

$$\text{AN} \quad \Leftrightarrow \begin{cases} X_A = 0 \text{ N} \\ Y_A = -5,49 \text{ kN} \\ Y_B = 13,49 \text{ kN} \end{cases}$$

$$\begin{aligned} q_1 &= 20 & L_1 &= 3,143 \\ q_2 &= 16 & L_2 &= 2,157 \\ F &= 8 \end{aligned}$$

Superposition:

$$\bullet \sum_{i=1}^3 Y_{Ai} = q_1 \frac{L_1}{2} + q_2 \frac{L_2^2}{2L_1} - F \frac{L_2}{L_1} = \frac{q_1 L_1^2 + q_2 L_2^2 - 2FL_2}{2L_1}$$

$$\text{AN} = 37,78 \text{ kN}$$

$$\begin{aligned} \bullet \sum_{i=1}^3 Y_{Bi} &= q_1 \frac{L_1}{2} + \frac{2L_1 L_2 q_2 + L_2^2 q_2}{2L_1} + \frac{F \times (L_1 + L_2)}{L_1} \\ &= \frac{q_1 L_1^2 + 2L_1 L_2 q_2 + L_2^2 q_2 + 2F \times (L_1 + L_2)}{2L_1} \end{aligned}$$

$$\text{AN} = 91,27 \text{ kN}$$